

Chapter 8 Periscope Antenna Systems Paul Wade W1GHZ ©1998,2000

8.1 Introduction

After we take great pains to optimize our antennas, we would like to mount them in a high place with a clear path in all directions. A good place is on top of a mountain — microwave operation from

mountaintops is very effective and great fun in summer. In winter, however, most of our mountaintops are inaccessible, and all are inhospitable. More importantly, we are rarely on a mountain during short-lived propagation enhancements. We would all like to be able to operate from home, with tower-mounted antennas, but many of us don't have super locations — my house is surrounded by trees, and my tower barely reaches the treetops.

Traditionally, we use a transmission line feed to get signals to the antenna. Feedline loss is a difficult problem for microwave antennas. Low loss feedline is expensive, and tends to be large in diameter. At 10 GHz, coaxial feedline larger than about a half-inch diameter will support waveguide modes that increase loss, so coax is not a good choice. Waveguide has lower loss than coax, but not good enough for a decent tower: the loss approaches 10 dB per 100 feet of WR-90 waveguide at 10 GHz.



Periscope Antenna System with Offset Parabolic Dish

One alternative is to mount parts of the system on the tower. Many hams have been using this approach successfully, but there are problems with weatherproofing and stability over temperature extremes. Tales of climbing a tower during a New England winter for repairs make this approach sound less attractive.

For several years now, Dick, K2RIW, has been talking about the merits of a periscope antenna system for microwaves. He convinced me to do some reading¹. Then I made some performance estimates for a reasonable size system. The numbers looked good, so I decided to put a periscope system together and see if it really worked. Initial trials showed that the periscope antenna really does work. Then, in order to improve the system, a better understanding was desirable.

8.2 Description

A periscope antenna system consists of a ground-mounted antenna pointed up at an elevated reflector that redirects the beam in a desired direction. A simple version, with a dish on the ground directly under a flat 45° reflector, is shown in Figure 8-1. The flat reflector is often referred to as a flyswatter, and we will use that terminology. The lower antenna does not have to be under the flyswatter reflector; the reflector tilt angle can compensate for offset configurations, as shown in Figure 8-2. The geometry is a bit more complicated, but a personal computer could easily do the calculations and control the flyswatter pointing.



Periscope antenna systems have been used for fixed microwave links with good results, but are no longer allowed by the FCC for new commercial installations. The reason seems to be that most good sites are so crowded with antennas that low sidelobes are required, and stray reflections from edges and supports of the flyswatter reflector make it difficult to meet the sidelobe requirements. However, for amateurs, antenna selection is a matter of individual choice.

The only recent publications found in major databases on periscope antennas are in Russian. In English, there is a description in the *Antenna Engineering Handbook*^{1,2} which relies on a confusing graph for explanation. After some study and working out a few examples, it dawned on me that

the graph is attempting to display an equation with four variables in a two-dimensional media.

The only amateur reference³ I've seen for periscope antennas is by G3RPE, and W1JOT was kind enough to provide a copy. G3RPE limits his analysis to 10 GHz, thus eliminating one variable, and provides six additional graphs to illustrate some of the possible combinations. This is an improvement but still doesn't provide much intuition. However, these curves and the ones in *Antenna Engineering Handbook* were enough for me to make some estimates and construct a periscope antenna system to demonstrate that it really works and is worth further effort.

8.3 Analysis

Lacking intuition, we must do things the hard way, starting with a mathematical analysis. Then we can try and present the results in a way that allows us to visualize what is happening.

There are two old papers on periscope antenna systems referenced in the *Antenna Engineering Handbook*^{1,2}, but I was not able to locate copies until several months after initial construction of my periscope antenna. The first paper, from 1953 by Jakes⁴, used an analog computer for the analysis which produced the confusing graph in *Antenna Engineering Handbook*. The second paper, from 1954 by Greenquist and Orlando⁵, proved more promising, since it included not only a more detailed analysis of periscope gain, but also some measured results from actual antennas. The gain at 4 GHz is presented as a series of graphs similar to Figure 8-3. These curves are for a flyswatter with a square aperture, but a circle or other shape would only change the gain by a dB or so. For simplicity, we will refer to the length of one side or the diameter as the flyswatter aperture dimension, and not quibble about that last dB.





A simple view of a periscope antenna is to consider it as a reflector antenna, just like a dish. The flat flyswatter reflector can be considered as a parabola with infinite focal length, so it must be fed from infinitely far away so that the illuminating energy is a plane wave. Far-field radiation from an antenna is approximately a plane wave, so the feed dish can provide a plane wave if it is far enough away so that the flyswatter is in the far field, or Fraunhofer region, of the dish — that is, beyond the Rayleigh distance. The gain of the periscope would be due to the aperture of the flyswatter, if the dish were able to illuminate it efficiently; thus, a larger flyswatter would provide more gain.

[There are also periscope antennas with parabolic-shaped flyswatters; we can consider these to be simply offset-fed dishes with a very long focal length, and analyze them accordingly.]

However, as we shall see, the periscope antenna provides much better performance if the reflector spacings are smaller, less than the Rayleigh distance of one of the reflectors. As a result, a more complicated analysis is required to calculate the gain. We must account for not only the imperfect illumination of the flyswatter, but also the path loss, or space attenuation, between the dish and flyswatter. Since path loss is defined between two isotropic antennas, we must also include the gain of the dish and flyswatter; both must be compensated for operation in the near-field, or Fresnel region. In the Fresnel region, the flyswatter may be illuminated with more than one Fresnel zone, and the second zone will be out of phase with the first, causing losses. In total, five terms are necessary for the periscope gain calculation:

- G1 the gain of the dish, with Fresnel correction factor
- Space attenuation the path loss between dish and flyswatter
- **G2** the aperture gain of the flyswatter when intercepting power from the dish, including Fresnel correction factor
- Edge effect loss of efficiency at the edge of the flyswatter due to diffraction
- **G3** the aperture gain of the flyswatter radiating into free space, corrected for illumination taper

The system gain of the periscope antenna is the sum total of these gains and losses.

The periscope gain calculations involve a couple of difficult functions, so it took some work before I was able to do the calculations. One function, the fresnel sine and cosine, we had needed previously⁶ to calculate the phase center of horn antennas; the code for this was written by Matt, KB1VC. To correct for the illumination taper, a spherical Bessel function, Λ , is needed. This function proved more difficult; the paper says, "Spherical Bessel functions of the form $\Lambda_p(\mathbf{v})$ are tabulated." For these functions, Silver⁷ refers to *Tables of Functions*⁸ by Jahnke and Emde; fortunately, Byron, N1EKV, had a copy and lent it to me so that I could make and verify the calculation. Today, of course, we have personal computers, so no one uses books of tabulated functions. [Note: the Spherical Bessel functions $\Lambda_p(\mathbf{v})$ are not the same as Bessel functions for spherical coordinates $j_n(\mathbf{x})$ found in modern references^{9,10,11}.] Once I was able to calculate periscope gain and reproduce the results of Greenquist & Orlando, I wanted to understand the complex relationship between the different dimensions. The equations are far too difficult to offer any insight, so my approach was to graph the results to try and visualize the relationships.

The first step was to replot the curves from Greenquist & Orlando in terms of wavelengths, so they are usable at any frequency. Each curve in Figure 8-3 shows the periscope gain for a specific dish size as a function of flyswatter aperture and height (reflector spacing). The gain of the dish alone is a horizontal line in the graph, and some of the periscope gain curves have portions, displayed in green, where the periscope system gain is higher than the gain of the dish alone. We can see that it is possible with some combinations to achieve a system gain several dB higher than the gain of the dish alone. Best gain occurs when the flyswatter is larger than the dish, and there appears to be an optimum height — but we can't see what produces the optimum. However, we know that path loss follows an inverse-square law: doubling the distance increases path loss by 6 dB. To increase reflector gain by 6 dB requires a doubling of aperture diameter. Thus we can understand why large gains require a large dish and larger flyswatter.

We can also see that large gains also require extremely large reflector spacing. Rearranging the curves for specific flyswatter sizes, in Figure 8-4, shows the same trend but doesn't really add any insight. The green portions of the curves again represent combinations where the system gain is higher than the gain of the dish alone.











Height (reflector separation) in $\boldsymbol{\lambda}$

Figure 8–4d – System Gain of a Periscope Antenna with a Flyswatter Aperture of 64 λ



The problem with these graphs is that we are trying to display a four-dimensional problem in a two-dimensional medium. A three-dimensional graph might help, if we could reduce the problem to three dimensions. My approach is to normalize the other quantities in relation to the dish diameter, so that one axis is the ratio of flyswatter aperture to dish diameter, and the gain is the effective gain of the periscope, the ratio of system gain to dish gain. The height, or reflector spacing, is normalized to the Rayleigh distance of the dish diameter D:

Rayleigh distance =
$$\frac{2D^2}{I}$$

In Figure 8-5, we can see the effective gain, or increased gain provided by the periscope system over the dish alone, increasing as the relative flyswatter aperture increases. The 3D plot in Figure 8-5a also shows that the range of optimum combinations is narrow and gain falls off quickly if we miss. The maximum effective gain shown is about 4 dB, with gain still increasing at the edge of the graph. The graph from Jakes⁵ shows an asymptotic value of 6 dB as the flyswatter becomes infinitely large. Other numerical values are difficult to discern from the 3D plot, so a 2D version is shown in Figure 8-5b, looking down from the top. We must rely on coloring from the gain bar to read to find effective gain values.

In Figure 8-5, it is apparent that the gain is not limited by the Rayleigh distance of the dish. However, if we instead normalize the height to the *flyswatter* Rayleigh distance, using the flyswatter diameter or square side for D in the calculation, the optimum combination becomes apparent in Figure 8-6. The 3D plot illustrates that the range is narrow, and the contour lines below the plot indicate the values. The contour lines for high effective gain are all in the range of 0.2 to 0.3 on the horizontal axis, the ratio of height to flyswatter Rayleigh distance. Thus, the height for best effective gain is roughly 1/4 of the flyswatter Rayleigh distance, regardless of the flyswatter size. Usually, we already have a tower of height **h** and would like to find the optimum size flyswatter aperture **A**:

$A \cong \sqrt{2hl}$

A flyswatter larger than this optimum size suffers excessive losses due to the Fresnel and illumination taper effects; in the plot, this is the area to the right of the gain peak. To the left of the peak is the area where the distance is too large or the flyswatter too small, and the gain decrease is inverse-square, due to space attenuation, the path loss between the two reflectors.

Since the Rayleigh distance is a function of the square of the aperture, the relationship between the dimensions is still not obvious. A realistic example should help. My periscope installation is about 20 meters high, or roughly 700 λ at 10 GHz. The effective gain for this height is plotted in Figure 8-7 as a function of dish and flyswatter size. The optimum flyswatter size is roughly 40 λ , about what we would calculate using the formula above. However, the effective gain increases as dish diameter decreases (note that this axis is reversed for better visibility). Yet we know that dish gain increases with diameter — so what is happening to the system gain? Look at Figure 8-8: the system gain has a



Figure 8-5b Periscope Antenna - Normalized Performance



Figure 8-6 Normalized Periscope Antenna Performance Height (reflector separation) vs. Flyswatter Aperture





broad peak with a dish diameter around 30λ and a flyswatter aperture around 40λ . If we can't find these exact sizes, any combination inside the contour circle below the 3D plot will be within a couple of dB of optimum.

What we learn from Figure 8-7 and Figure 8-8 is that a smaller dish contributes only a small gain to the system, while a larger dish is too large to illuminate the flyswatter effectively at this distance. Best system performance is a compromise between the effective gain of the periscope and the gain of the dish alone, and must be determined for a particular height.

The effect of height, or reflector spacing, is illustrated by Figures 8-9 through 8-12, showing system gain for a range of heights from 250 to 1500 wavelengths — when we work in wavelengths, the results are applicable at any frequency. For each height, there is an optimum combination of dish and flyswatter sizes; more important, there is a maximum gain for each *height*, with larger gains possible for greater reflector separations.







Figure 8-12 System Gain of Periscope Antenna Height (reflector separation) = 1500).



42

The beamwidth of the periscope antenna is controlled by the aperture of the flyswatter reflector. The beamwidth is nearly independent of the gain, just as we saw for dishes in Appendix 6A. Thus it is possible to have a flyswatter with low gain and a narrow beamwidth, if the dimensions are far from the optimum area found in the graphs.

Reaching an intelligent compromise requires that we be able to estimate performance. The graphs included here should be adequate for rough estimates; they were created using **MATLAB**¹² software, which is powerful but a bit expensive for amateur use. However, I was able to create a Microsoft Excel spreadsheet that performs the periscope performance calculations, so that you may make accurate estimates for any dimensions. You may download **periscopegain.xls** from <u>http://www.qsl.net/n1bwt</u>.

8.4 Summary

We can achieve reasonable antenna gain with a dish on the ground, so feedline loss may be very small. The "feedline" loss in the flyswatter is power radiated from the ground antenna that misses the flyswatter and continues into space; this is obviously dependent on the geometry of the periscope components. Some combinations of dish, flyswatter, and spacing can provide more gain than the dish alone — like a feedline with gain!

The beamwidth of the periscope system is similar to the beamwidth of a dish with the same diameter as the flyswatter.

For some combinations, this can result in a beamwidth significantly narrower than a dish with equivalent gain — a minor disadvantage for the periscope antenna.

Another minor problem with periscope antennas concerns polarization. If the flyswatter rotation is independent of the ground antenna, then the polarization changes with rotation. There are several ways to compensate:

- 1. Rotate the ground antenna as well as the flyswatter.
- 2. Rotate the feed polarization.
- 3. Use circular polarization and accept 3 dB loss in all directions.

For rain and snow scatter, my observation is that polarization does not seem to be particularly critical.

Since the flat flyswatter is not frequency sensitive, it can be used on other microwave bands as well. The ground antenna could be a dish with feeds for multiple bands, or separate ground antennas could be used, adjusting the flyswatter angle for each band.

8.5 Construction

As an example of a rotating periscope antenna system, I'll describe the one I built. I've seen commercial periscope installations with a fixed flyswatter and they look pretty straightforward: the flyswatter is attached to the side of the tower and bolted down after adjustment. However, there aren't any examples of a flyswatter that rotates and tilts, so the difficult part was figuring out the mechanics.

My house is surrounded by trees, and my tower barely reaches the treetops. Prior to starting on the periscope system, some tests had shown that 10 GHz operation is possible at my QTH. I have made rain scatter and snow scatter contacts pointing straight up through a skylight, and other contacts by aiming a dish at the tops of trees to scatter off them. If I could get the 10 GHz signal to the top of the tower, better results should be possible.

The first step in constructing a periscope was to find a good-sized piece of aluminum for the flyswatter. I located a 30-inch (0.76 meter) octagon left over from one of my daughter's high school adventures (I've been assured that it came from scrap somewhere and not from a road). To stiffen the mounting area, I attached a heavy aluminum frying pan with a flat rim to the center, and bought a new pan for the kitchen.

One approach for rotation would be to mount the flyswatter on the central mast below other antennas. The mast could pass through a central hole in the flyswatter. For a larger flyswatter, the area blocked by the tower would be small, but I wanted to do initial testing with a reasonable size reflector. An alternative shown in the G3RPE article³ is to mount the flyswatter on one side of the mast, with the ground antenna following it around the tower as it rotates.

A simpler approach is to mount the flyswatter on the side of the tower, with rotator and support above it and out of the RF path. Since I didn't trust an ordinary rotator in tension, with the antenna weight pulling it apart, I chose the style that has the mast passing through the body of the rotator. This is attached to a very solid side bracket, available from IIX Equipment¹³.



Tilting the flyswatter is a little more difficult, but is important for scatter propagation. For a pivot, I drilled a hole through the mast for a stainless steel rod which passes through sleeve bearing on each side. Two pieces of angle aluminum are mounted to the frying pan with a bearing fit in each; the mast is sandwiched between them. Figure 8-13 is a photograph of the assembled tilt mechanism which should make this clearer. To power the tilt mechanism, K2CBA provided an old TVRO dish actuator. The flyswatter trial assembly at ground level is shown in Figure 8-14.



When we tested the flyswatter before raising it, we found that the tilt actuator would bind up — the actuator was originally clamped in place by U-bolts, but it obviously needed to pivot as the flyswatter tilted. Another bearing was needed in place of the U-bolts. A quick trip to the hardware store located a large swivel caster, normally used for moving heavy machinery, which fit the U-bolt holes. When the caster wheel was removed, the actuator fit in the space and was clamped in by the axle bolt, as shown in Figure 8-15. Now the tilt operated smoothly, and we could move the assembly into position near the top of the tower, as shown in Figure 8-16.





8.6 On the air performance

For initial testing, I set up a 10 GHz rover system with an 18" DSS dish directly under the flyswatter. Separation between dish and flyswatter was about 10 meters, so the estimated gain from the curves was about 5 dB down from the 18" dish, but with the narrow beamwidth of a 30" dish. Also, only crude azimuth and elevation indication were available. Clearly, this was not the optimum configuration, but adequate for initial testing.

The first 10 GHz tests were disappointing. Without any rain or other propagation enhancement, distant stations were extremely weak, and closer ones were audible in all directions with no discernable peak. With trees in all directions, wet foliage was scattering and absorbing signals. After a couple of days without rain, signals weren't much better. The flyswatter does not clear the treetops, and foliage has significant attenuation at 10 GHz.

Rainscatter performance is much better. With rain predicted for the 1998 June VHF QSO party, I made a radome over the 10 GHz rover system using clear plastic garbage bags. We had several inches of heavy rain, which produced strong rainscatter signals. I was able to work stations in four grids, with best DX of 211 km., and probably could have worked more if the rain hadn't stopped on Sunday morning. Most of the contacts were on CW, but AF1T, 66 km. away, was so loud that we switched to SSB. If you'd like to hear Dale's signal, as well as some other rain and snow scatter signals, there are some sound clips at <u>http://www.qsl.net/n1bwt</u>.

Rainscatter signals typically have fairly broad headings, so my crude azimuth indication was adequate. For elevation, a local beacon peaked broadly somewhat above horizontal, so I left the tilt at that setting. For normal propagation, beamwidth of the periscope antenna should be quite narrow, like any high gain antenna, so a better readout system is needed.

Since these initial tests, I have added a larger fixed dish at the base of the tower, shown in Figure 8-17, and a digital tilt indicator. With these improvements, I am able to make local10 GHz contacts, plus more distant ones, typically in three grid squares, whenever there is any precipitation. Rain scatter has an auroral quality, probably from random doppler shift from raindrops falling at different speeds. Snow scatter can provide outstanding signals — if there is no wind and the snowflakes are large, the flakes fall slowly and good SSB quality is possible. Unfortunately, the northeast part of the USA had a severe drought for most of 1999, so good conditions were rare.

One last point concerns wind loading: I have never seen the flyswatter move or waver in any wind condition, even though the rotator is a cheap TV model with plastic gears. A dish of the same diameter has much more wind load than the flat reflector and would probably have long ago stripped the gears. For a very large flyswatter, it might be prudent to tilt it flat when not in use to further reduce wind load.

8.7 Enhancements

The original periscope trial system described above was rather small; both the dish and the flyswatter are undersize for 20 meters separation, resulting in the estimated 5 dB loss. One solution would be to reduce the separation, but this would elevate the bottom dish and require a lossy feedline. A better solution would be to increase the size of the dish and flyswatter.

I recently increased the dish size and made the installation more permanent by adding a fixed one-meter offset-fed dish at the base of the tower, shown in Figure 8-17. The flyswatter mounting structure was designed to accommodate a flyswatter at least 1 meter wide and 1.25 meters high, so there is room for improvement here also. Also, since dishes and flat reflectors aren't frequency sensitive, it would be great to use the periscope system on more than one band. The calculated periscope system gain in Figure 8-18 shows roughly 3 dB improvement at 10 GHz with the one-meter dish over the dashed line for the previous 18" dish.

Figure 8-18 also shows the system gain for several microwave ham bands as a function of flyswatter aperture. Clearly, a larger flyswatter than the current 0.76-meter aperture is desirable, particularly at lower frequencies. The optimum flyswatter aperture for 10 GHz is about 1.2 meters, which is not unmanageably large. Figure 8-19 plots the periscope system gain vs. frequency for the one-meter dish with a 1.2-meter flyswatter as well as the current 0.76-meter one with both the one-meter



dish and the original 18" dish. The dotted curves show the gains of the dishes alone. The current system is comparable to a 24" dish at 10 GHz and falls off at lower frequencies, but the largest combination provides performance close to a one-meter dish, not only at 10.368 GHz, but also at 5760 and 3456 MHz. This looks like a winner! Even at 2304 MHz, the mediocre gain is comparable to a medium-sized loop Yagi.





These additional bands are almost free — only a feedhorn is required. Two-band operation is simple: a dual-band feedhorn will do the job — several variations are described in Chapter 6-9. An offset dish has a huge advantage for multiband operation: the feedhorn and support are out of the beam. I envision a carousel of feedhorns next to the dish, changing bands by rotating the appropriate horn into position at the focal point of the dish. One way to do this is with an ordinary antenna rotator, like the arrangement by KA1ZE in Figure 8-20. A more complex arrangement, by WD4MUO¹⁴ in Figure 8-21 (another view may be found in Figure 6.9-35), moves the desired feedhorn into position and simultaneously connects it to an appropriate waveguide output.



Will the periscope work on higher bands? Of course it will, if the dish and flyswatter reflectors have surfaces that are good enough — parabolic or flat within about $1/16\lambda$. As an example, I made some estimates for my system at 24 GHz. For a height of 20 meters, our formula estimates an optimum flyswatter aperture **A** of 0.7 meters, close to my current system, so I extended the curves of

Figure 8-18 to include 24 GHz in Figure 8-22. The curve for my current combination, a one-meter dish with a 0.76-meter flyswatter, has maximum gain around 15 GHz and falls off by 24 GHz to provide less gain than at 10 GHz. My proposed larger flyswatter is even worse at 24 GHz, with low gain and very narrow beamwidth due to the large flyswatter aperture, a bad combination. However, the



smaller flyswatter with a small 0.5-meter dish looks good — the gain is higher than the dish alone. Feedline with gain at 24 GHz is a real miracle!

As usual, we can't have everything, and must compromise. With the larger dish and flyswatter, I could get good performance on 10 GHz as well as 5760 and 3456 MHz. With the smaller dish and flyswatter, I could have 10 GHz and 24 GHz but not much gain on the lower bands. Since my biggest obstacle is foliage, I would probably do better at lower frequencies. However, a more complicated alternative might be to use the smaller flyswatter with both dishes, moving and tilting the flyswatter to change bands.



8.8 Conclusion

The periscope antenna system is worth considering as an alternative to high feedline losses or tower mounted systems. The analysis and graphs in this chapter should enable you to understand periscope antenna operation and allow you to design one with confidence.

My periscope antenna system has enabled me to achieve better microwave results from the home QTH than I could before — without it, I can hardly get out of my own backyard if the weather is good. The system works well on rain and snow scatter, and there are many locations with more rain than altitude. I don't believe I have explored the full potential of this antenna, so I urge others to try it and report the results.

Finally, special thanks are due to Rick Campbell, KK7B, who helped me overcome a fear of Bessel functions.

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